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Book review

GROUP THEORY and PHYSICS by S. Sternberg; published by Cambridge University Press *

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This is a thought-provoking work with a cogent viewpoint and an incisive style. Its dual aims, of seeking to make the somewhat more specialist mathematical, combinatorial and particle structure physics literature accessible to a wider physical science audience, and of giving the mathematics postgraduate a broad appreciation of physical applications of various areas of mathematics associated with group theory, are admirable and distinguish it from existing texts and monographs [1] which root themselves firmly in one or other discipline. The author is a well-known Harvard mathematician, with group theoretical interests that, in the main, focus on nuclear particle structure; however, the text is considerably broader than this specific interest. It does not discuss the most recent work beyond 1986, on which there remains any sense of incomplete assimulation, or residual unresolved open questions, though there are valuable additional comments on the way forward in the particle structure area. Likewise, it does not discuss a number of prominent areas which are of recent origin, such as superstring theory and quantum groups. The researcher in molecular physics will not find any discussion of the wreath-product groups, of the power of plethysms in handling tensors products (though this may be found in the James and Liebeck mathematical text [2] on representations of groups), or of the determinacy of embedded finite groups in highly branched $\mathcal{S}_n$ spin symmetries. More central to physics applications of group representation theory, there is no mention of any specifics (e.g., of the role of Schur functions in subduction) of the group chains underlying cluster spectra, despite a passing reference to Euler equation within the geometric basis of fullerene structures.

To give the prospective reader a firmer outline of the main topics covered, we shall paraphrase some of the points of special focus in the work. The view adopted in setting out the basic material is taken from the group actions on a set and concomitant ideas of the stabiliser subgroup. The finite subgroups of O(3) are introduced using illustrations drawn from crystal structure classification and also the concept of covering groups, prior to a character-based discussion of representational theory of finite groups. The author's comments on both fixed-point formulas and aspects of $\mathcal{S}_n$ groups are

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amplified in a number of valuable appendices. The appendix on combinatorial aspects of $\mathcal{S}_n$ representation theory constitutes an especially valuable summary of an area not well-covered in physics-based texts.

Chapter III discusses various types of treatment, and the selection rules, of molecular vibrational spectra; in contrast to the works of James and Liebeck, and others, the concept of vector bundles is introduced to provide physical intuition to a valuable overview of induced representation, a topic discussed in by Coleman in a more specialist monograph [1] in the mid-1960s. After some discourse on the Frobenius semi-direct product representations and the use of tensor products in the context of selection rules in quantum mechanics, this chapter concludes with an incisive discussion of the discrete symmetries of space–time, governing parity and time-reversal. This should be invaluable to the chemical physicist or NMR specialist, for whom the latter are now primary topics of recurrent general interest.

Chapter IV on compact groups with pertinence to the role of spherical harmonics in atomic and nuclear shell theory leads naturally into treatments of the hydrogen atom and of the role in physics of magic numbers. The presentation here is admirably concise. The reader is left with the strong impression that the mathematics and theoretical physics contributions are essentially indistinguishable in the way they provide an interpretation of these fundamental aspects of physics. Subsequent sections of the work discuss Clesch–Gordan coefficients, as applied to isospin in the context of pion-nucleon scattering, and a number of standard topics of theoretical physics, such as the Klein–Gordan and the Dirac equations, and how the relativistic wave equation relates to the Wigner classification of the appropriate irreps of the Poincare group. The final section gives a useful, if brief, summary of the methods of Lie algebra.

Chapter V reviews the classic area of Schur–Weyl duality between the $\mathcal{S}_n$ and general linear group, as a prelude to a discourse on weight-vectors of the SU(m) groups. The discussion then moves on to consider various classifications of reaction channels pertaining to strong and weak interactions, before presenting the elements of quark structure inherent in baryons and mesons, given here one notes in terms of tensor products and their associated group chains, and some discussion of nucleon magnetic moments in terms of quark theory.

Within the author's time-frame (up to the mid-1970s/early-1980s) and his focus on resolved questions, the work is a valuable synthesis of conceptual ideas and a highly readable contribution to the theoretical interdisciplinary science literature. Its value for those in the fields of mathematical and quantum chemistry lies in the breadth and depth of topics covered with which one should be reasonably familiar. I particularly enjoyed an appendix on the combinatorial aspects of group theory, including a number of proofs of results in representation theory of the $\mathcal{S}_n$ group, i.e., at a less specialist level than given in Sagan's 1991 mathematics monograph. The author is to be congratulated for the way in which he conveys a sense of enthusiasm for the subject-matter to the reader, and for his inclusion of a valuable bibliography; the publishers are also to be praised for the attractive format of the book, essentially free of misprints. Finally for inclusion in one's personal library, the paperback price is modest, so that the work will be an

admirable complement to various standard works for anyone with a serious interest in group theory applied to the physical sciences.

References (additional to those cited in the reviewed text)

[1] A.J. Coleman, *Induced Representations on* $\mathbb{C}$ *for* $\mathcal{GL}(n)$ *and* $\mathcal{S}_n$ *Groups* (Queen's Math. Publ., Kingston, 1964).
[2] G.D. James and M.W. Liebeck, *Representations and Characters of Groups* (Cambridge University Press, 1993).